A Powerful tool

Some frequency analyzers can be built right into controllers. They should be a top consideration when choosing a motion controller for complex system applications — as frequency analyzers can be used to design for servo systems with low mechanical stiffness.

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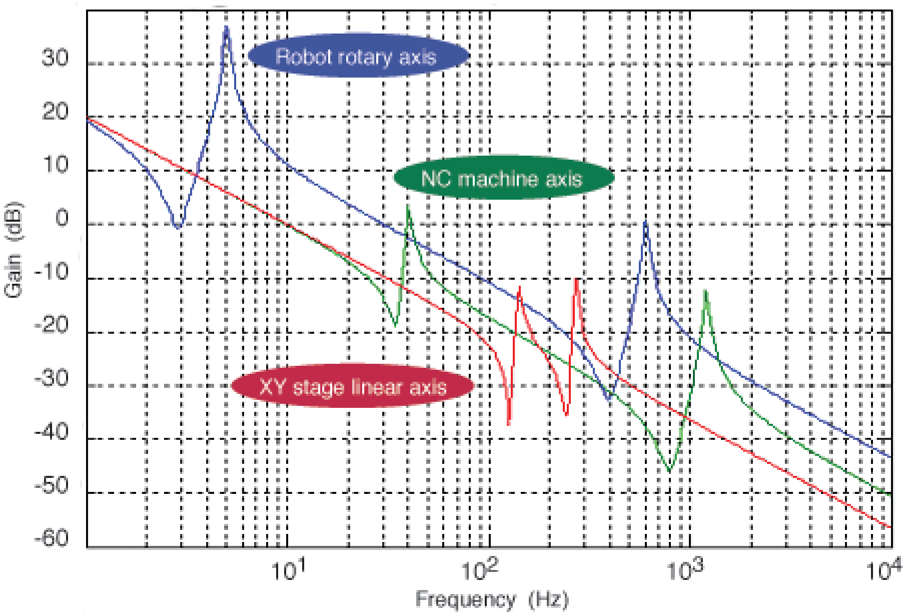
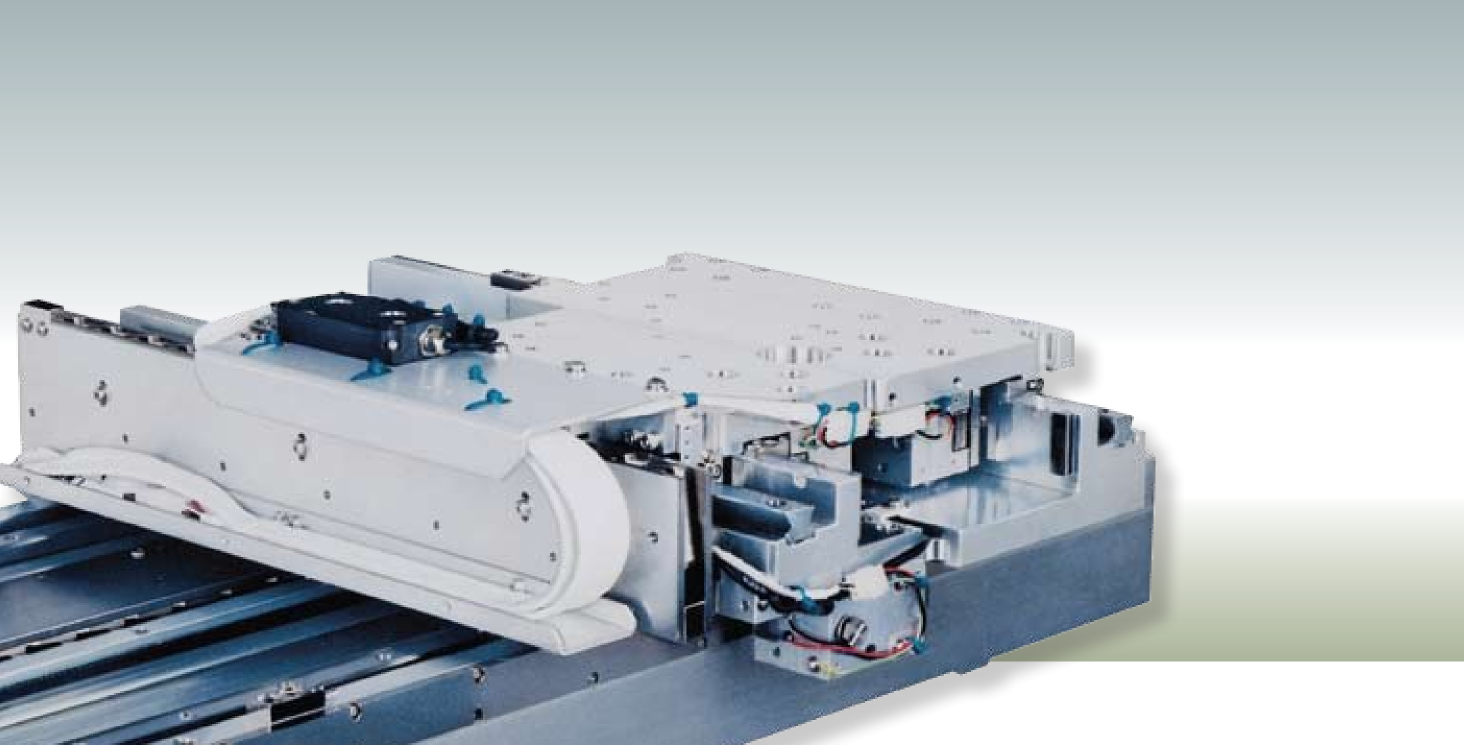
Does your servo system exhibit tremors or unsteady motion — say, during drilling, wrapping, or other functions? You may be looking to optimize performance despite low mechanical stiffness. If so, consider a motion controller with a built-in frequency analyzer at redesign. Here's why: Frequency analyzers are increasingly available from several manufacturers (ACS, Galil, MEI, Mega-F, Control and Robotic Solutions, and Siemens, for example) and are now advanced enough to be integrated into motion controllers. What is more, newer software puts their capabilities to higher work.

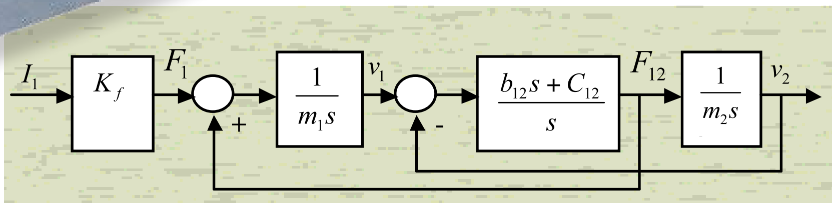
The capability to accurately measure a system's mechanical resonance frequency allows one to create a mathematical model for the system. Using this model, designers can simulate and calculate the system's dynamic motion response and make real-time adjustment in the motion controller — to minimize or eliminate instability caused by mechanical resonance, as well as optimize dynamic system performance. Modern state-of-the-art motion controllers allow creation of both useful and accurate frequency analyzers over a range of frequencies. A wide bandwidth (not less 1 to 2 kHz)

Approximated experimental Bodes





Current loop model



This current loop model for a two-body linear system assumes totally linear behavior. I_1 is the motor current command, F_1 and F_{12} are motor and load-to-motor forces; $v_1(s)$ and $v_2(s)$ are motor and load velocities, K_f is force constant; m_1 and m_2 are motor and load masses; c_{12} is load-to-motor linkage stiffness and b_{12} is the internal friction constant.

in a servo amplifier (pulse width modulated or linear) is advisable for accurate identification of each resonance frequency. For systems with low mechanical stiffness in the linkage or transmission connecting the load to the motor, a frequency analyzer is especially important due to the low-frequency (0.5 to 10 Hz) mechanical resonance.

Axes with low mechanical stiffness

Servo systems generally have more than one resonant frequency. Even so, usually only one resonant frequency dominates dynamic system performance and stability within the frequency range defined by the servo controller bandwidth. Then again, a resonance outside the controller's bandwidth can significantly limit the maximum allowable system bandwidth.

These phenomena can be explored by modelling a system as multiple discrete masses or bodies interconnected by low-stiffness linkages. For example, a simple two-body mechanical model and the superposition principle approximates a mechanical system of three bodies and two linkages. Let us consider some of the key features of this basic two-body mechanical model.

Real servo systems are typically nonlinear, but when modeling them, mechanical-linkage backlash, internal viscous damping, and other nonlinearities are generally ignored.

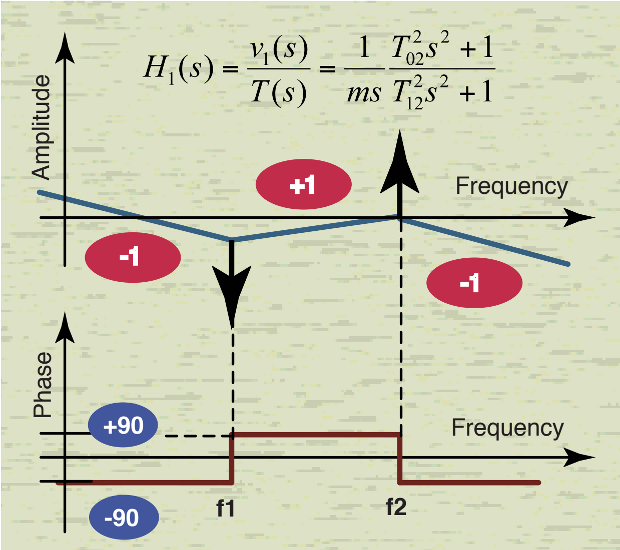
When creating a linear model for a non-linear system, assume:

- Motor and load mass are concentrated and nondeformable
- Rigidity of low-stiffness linkage is considered constant
- Damping of resonance vibrations is due to internal friction of linkage only
- Clearance and backlash in linkage are zero
- Disturbance forces from direct axis and cross-coupled axes are neglected

The fairly well known current-loop model for a two-body linear system is based on these assumptions. The asymptotic

Servo systems (including CNC machines, robots, XY stages, and so on) often exhibit more than one resonance frequency. Many mechanical systems can be modeled as three bodies and two linkages: For example, Mega-F software shows that the first (or lower) resonance of a CNC machine axis is due to low stiffness in the linkage between the ballscrew and the table, and the second resonance is due to low stiffness in the linkage between motor and ballscrew. Hence, basic two-body mechanical models are common.

Motor: Asymptotic Bode plot



A motor's motion response is shown in this Bode plot of motor mass for our current loop with low stiffness. It corresponds to the transfer function *H1(s)*.

Bode plots of motor mass for this current loop with low stiffness — without taking into account damping actions of internal friction b_{12} — corresponds to the transfer function:

$$H_1(s) = \frac{v_1(s)}{T(s)} = \frac{1}{ms}\frac{T_{02}^2 s^2 + 1}{T_{12}^2 s^2 + 1}$$

Where $m = m_1 + m_2$ is total mass and $Y = m_2 \div m_1$ is the mismatch ratio. In addition, the time constant for two-body elastic vibrations is:

$$T_{12} = \sqrt{m_1 \cdot m_2 \cdot c_{12}^{-1} \cdot (m_1 + m_2)^{-1}}$$
$$= (2\pi \cdot f_2)^{-1}$$

The time constant of second body's vibrations with stiff attachment of motor's moving coil or magnet is:

$$T_{02} = \sqrt{m_2 \cdot c_{12}^{-1}} = (1+Y) \cdot T_{12}^2$$
$$= (2\pi \cdot f_1)^{-1}$$

Analysis of the transfer function $H_1(s)$ and its relation to the Bode plots reveal significant two-body linear system features. Specifically, the motor only displays both anti-resonance frequency f_1 and resonance frequency f_2 as shown in asymptotic Bode plots of the motor's behavior. Antiresonance frequency f_1 is independent of motor mass and sets up the system such that its total momentum of vibrating system equals zero at resonance frequency f_2. Resonance frequency f_2 is caused by the connection of the two masses through a linkage with finite stiffness C_{12}. Antiresonance frequency f_1 is always lower than resonant frequency f_2, and differs from f_2 by factor (√1+Y) as well. These features of the motor's Bode plot allows calculation of system parameter values with defined total mass m such that:

Mismatch $Y = (f_2 \div f_1)^2 - 1$

First resonance mass $m_1 = m \div (1+Y)$

Second resonance mass $m_2 = m - m_1$

Linkage stiffness $c_{12} = (2\pi \cdot f_1)^2 \times m_2$

Key analyzer features

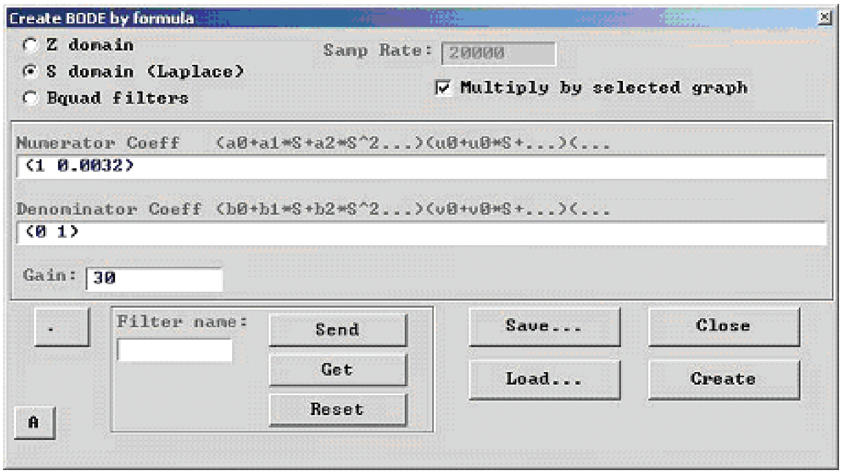
Modern motion controllers often allow for automatic or manual measurement of a servo system's open-loop Bode plot; in this way, they provide necessary gain, bandwidth, and phase margin. Some controllers can even measure a Plant Bode plot that also includes the power amplifier and motor.

The final measured frequency response can be presented in different formats, including Bode, Nyquist, or Nichols curves; experimental Bode plots can be saved in either TEXT or MATLAB format for both convenience and subsequent analysis.

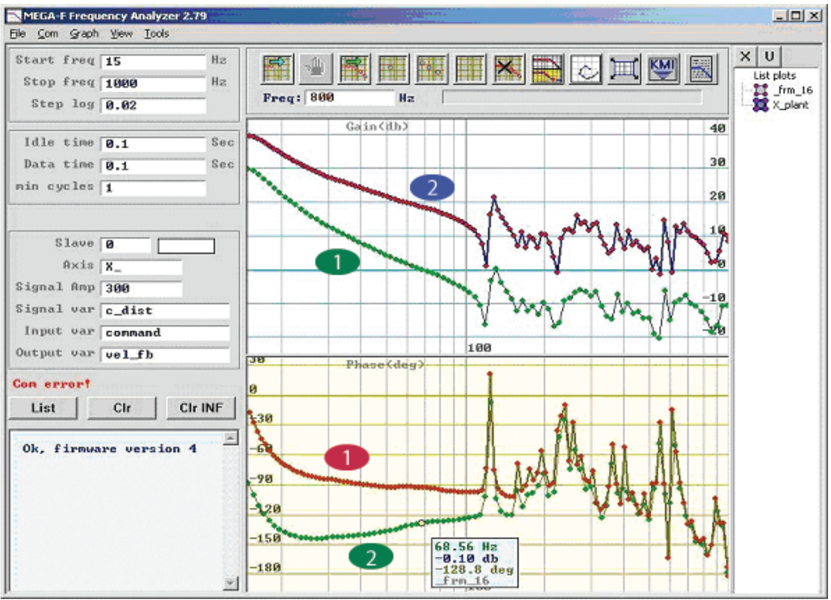
Design mode features

Except for the direct measurement of frequency response, it's often possible to use a special Design mode to measure system performance with different tuning options. This Design mode uses

Design-mode subwindow



Plant and open-loop Bodes



Over a test range, one precision linear axis has frequencies using both stepped input and sinusoidal excitation. On this linear axis, the XY stage Plant labeled ❶ provides the Bode plot with transfer function $H_1(s)$. Open-loop Bode plots (labeled ❷) can be used to check loop performance: Here, the text box reports a phase margin of 51.2° with crossover frequency (bandwidth) of 60.56 Hz.

measured Plant frequency response as its baseline. One standard control filtering technique provided in modern controllers (PIV as a rule) can be added to analyze closed-loop performance based on the system's open-loop frequency response. For this feature, some controllers allow uploading of control-filter parameters for the structure by inserting the transfer function for control filter with specific software. For example, the Design mode subwindow for certain software packages allows use of the experimental Plant frequency response as a baseline by using the plot as a selected graph.

Furthermore, a designer can insert the transfer function for the control filter into the numerator, denominator, or both. To illustrate, consider a PI control filter having the transfer function:

$$H_{PI}(s) = K_{PI}\frac{T_{PI}s + 1}{s}$$

Let us assume gain KPI = 30 and time constant TPI = 0.0032 sec are defined as shown in the **Design mode subwindow** on the previous page. This "standard" PI filter can be augmented by adding both low pass and notch filters.

These well-known filters dampen for both resonances and acoustical noise caused by low shaft stiffness between the encoder and motor, or between the motor and the ball or leadscrew — and shows up in the measured frequency response before activation of these added filters.

Example: Linear axis of XY stage

Let us now assume that we have a precision linear stage. The model of this component includes a Plant with a velocity loop that utilizes both input current command and motor output velocity as its feedback. Over a test range of frequencies using both stepped input and sinusoidal excitation of the linear axis, the XY stage Plant feeds the Bode plot with transfer function $H_1(s)$.

Based on our two-body mod-

Frequency analyzer comparison

Factor		ACS	Mega-F	CRS
Frequency domain characteristics ...	Plant	+	+	+
	control filter	+	+	-
	open loop	+	+	+
	close loop	+	+	+
	any units	-	+	-
Frequency response format	Bode	+	+	+
	Nyquist	+	+	+
	Nichols	-	+	-
Mode "design"	Low-pass filter	+	+	+
	Notch filter	+	+	+
	Any transfer function	-	+	-
	Servo performance auto calculation	+	-	+
Frequency range, Hz		1 ... 5,000	*	*
Saving data format	Graphic	+	+	+
	Text	+	+	+
	MATLAB	-	-	+

el, analyses of this plot show the presence of mechanical resonance inside the velocity-loop bandwidth. Each resonance exhibits strongly marked features with the presence of both resonance and anti-resonance frequencies along with associated phase discontinuities. To analyze velocity-loop stability using our example software package, we use the followings steps.

- Obtain Plant's experimental Bode plot, and select this Bode plot
- Open the Design mode subwindow and check *Multiply by selected graph*
- Insert control filter parameters as numerator and denominator coefficient in the S-domain; get open-loop Bode plots by pressing Create button
- Check loop performance, and check any points of Bode phase crossing (or -180°) for proper gain margins

Note that Design mode does not account for any reference (velocity and acceleration) feed forwards, disturbances (such as cogging and friction), or compensation for reducing of reference and disturbances errors.

GUI support

In addition to the above capability of advance motion analysis, certain software supports the option of designing a unique application and running frequency analyses as well as data collection utilities within the customer or end-user application. This option allows performing of automated routine that will move for example your axes to predefined locations and perform Plant Bode analysis on each location with automatic data saving in the output file.

The output file along with its frequency domain data can be used for further analyses. The benefit of having an automated Bode plot built directly into the motion controller is that it provides much tighter control of stage assembly along with pass-fail criteria during preproduction. The collected data can also be used to improve a system's mechanical design.

For more information, contact Mega-F Motion Systems Ltd., Smithtown, N.Y. at (631) 486-2043, or visit rockwellautomation.com/anorad.