Linear shaft motor SIZING

One of the most straightforward tasks in linear motion system design is to specify a motor and drive combination that provides the force, speed, and acceleration required by the mechanical design. All too often, it is the most overlooked.

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Even knowledgeable engineers can miscalculate required system force, often by selecting a motor, and then designing a system around it. This approach leads to motors that are overly large or small for an application, and can be the most costly aspect of a system design — in initial purchase cost and in service and energy usage.

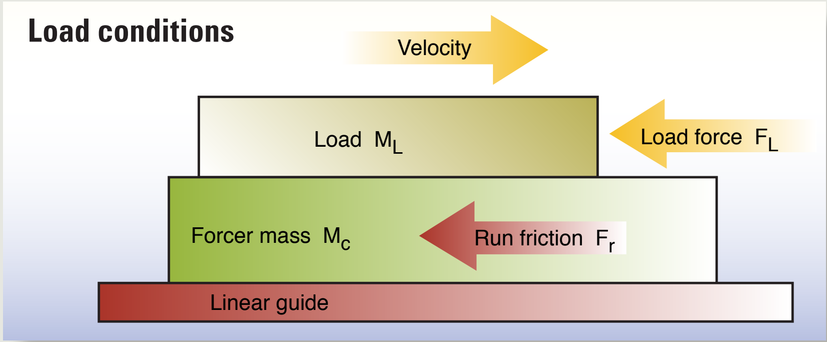
Inappropriate designs also happen most when designers incorporate specialized motion components into systems. Linear shaft motors are no exception; they size differently than other motors.

Let us review the basics of linear shaft motor sizing and the steps performed by this software to understand the process, and allow for calculation of even complicated motion systems — in four steps: First, define operation conditions and motion profile. Then calculate forces required by the system. Next, select the proper linear shaft motor for the application. Finally, select the proper linear servo driver.

Define conditions and profile

The first step is to define basic operation conditions. This includes load to be moved (load, table, encoder, external load forces, and friction coefficient), available power (current and voltage), and environmental issues, such as temperature and water. (Unless otherwise stated, all values in this article are rms.)

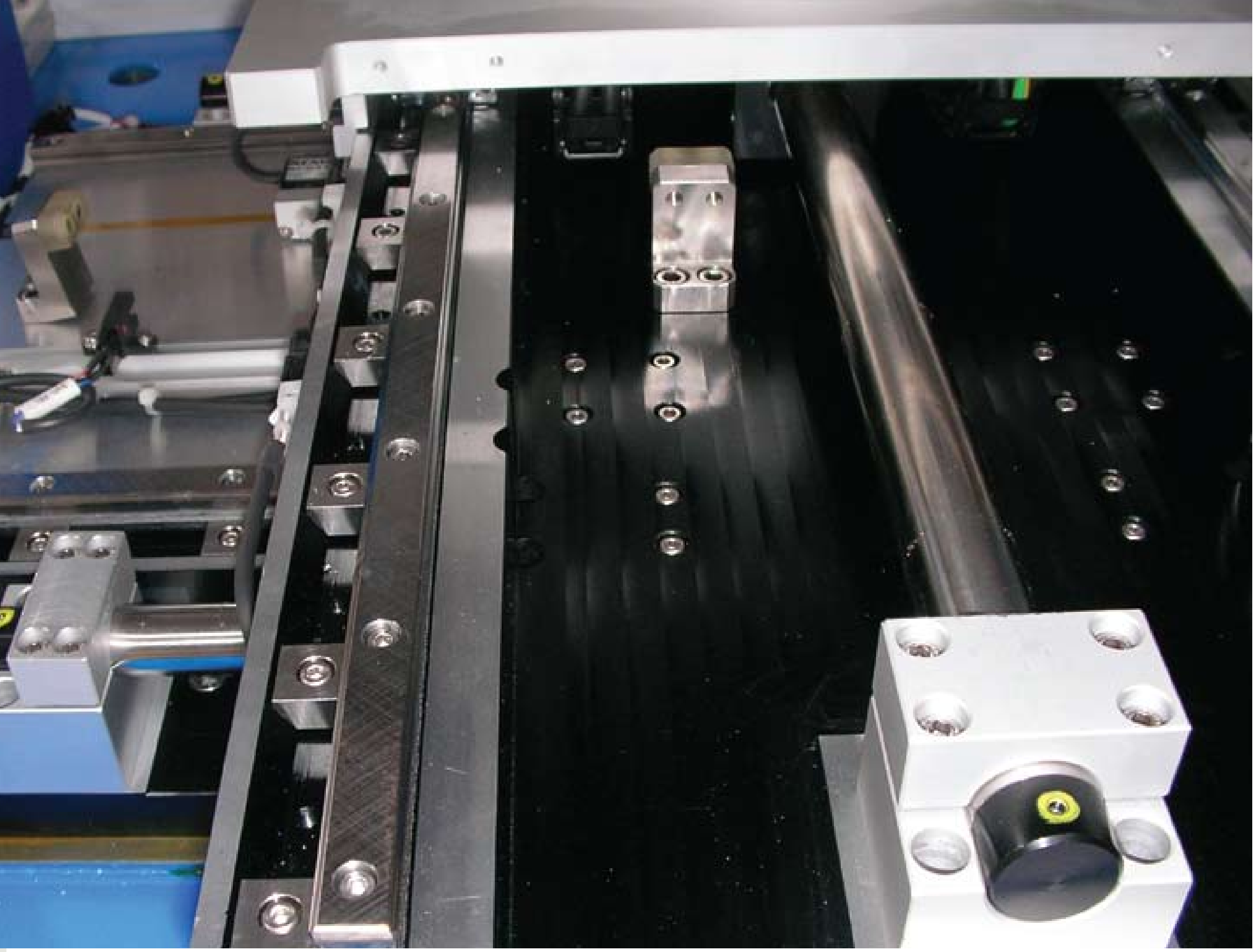
Next define system motion (if any) in terms of stroke, velocity, and time. Though all of this information may not be known at the start, more information makes for better motor selection. Let us work through two examples with each step.



More than just the load M_L must be considered when sizing an application. Run friction F_r includes the frictional load of the linear guide (also called preload), the resistance force of the cable carrier to motion, and any external dampening forces that are equal in both directions (for example, a system submerged in water or a gas shock) and all are treated as load forces.

Load force F_L includes both force that resists motion in one direction and assists in the other (as in a spring, for example) or a force that a motor exerts on an object — to push a box, for example. The only exception is gravitational force, which is always assumed in calculations.

Forcer mass M_c is that of a moving forcer, or the shaft in a moving shaft design. Tip: If unsure as to which linear shaft motor a design will require, use ¹⁄₁₀ load mass as the value for forcer mass M_c in calculations.



Example ➊: Assume that we have a pick-and-place application and it must horizontally move a 2-kg load point-to-point for 430 mm, with an acceleration of 1 *g* and a maximum velocity of 760 mm/sec. There will be a 3 sec dwell after each move.

Example ➋: Assume that we have a pin insertion application. Here, we want to vertically move a 3-kg load point-to-point for a distance of 50 mm in 120 msec. There will be a 10 msec delay for settling after the downward move, and 150 msec of dwell after the upward move.

There are three common motion profiles. Triangular ½-½ profiles accelerate to speed, decelerate back to original speed or zero, rest, and repeat as needed. This simple motion profile is common in pick-and-place applications.

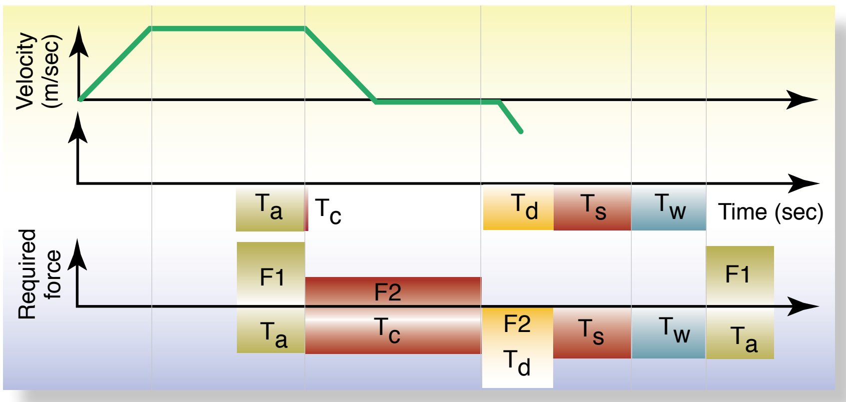
More sophisticated trapezoidal profiles accelerate to constant speed, travel at that speed, and *then* decelerate back to original speed or zero — common in applications such as scanning inspection.

Example ➊: Variable trapezoidal profile: V = 760 mm/sec and A = 9.81 m/sec^2 ... $X_a = X_d$ =0.7602 / (2 × 9.81) = 29.4mm ... Xc = 430 – (29.4 × 2) = 371.2 mm ... $T_a = X_a$ / (0.5×V) = 29.4 / (0.5 × 760) ... $T_a = T_b$ = 77 msec
$T_c = X_c/V$ = 371.2 / 760 = 487 msec
Example: ➋: ⅓ trapezoidal profile
T=120 msec and X = 50 mm
T_a= T⁄3 and $T_a = T_b = T_c$ = 40msec
V = 1.5 × 50 / 0.120 = 625 mm/sec

Settling and waiting time

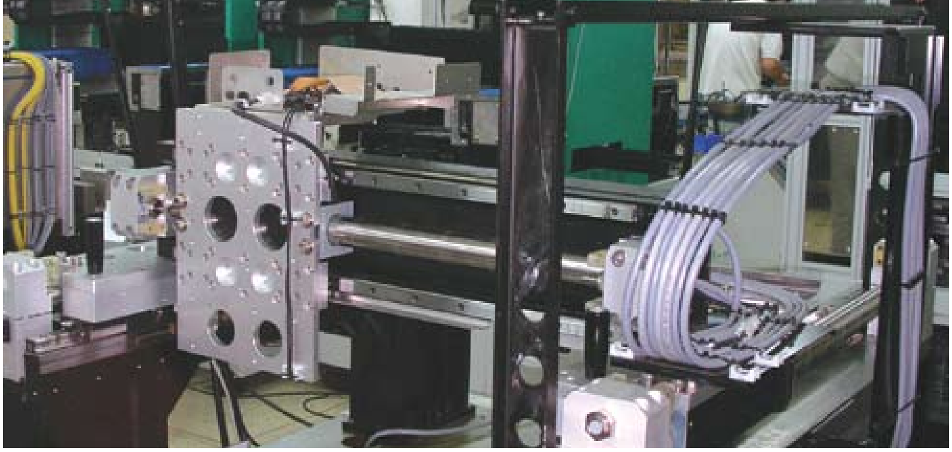
In a motion profile, waiting time T_W plus settling time T_S equals to-

Trapezoidal motion profile



There are two trapezoidal motion profiles: 1⁄3-1⁄3-1⁄3 trapezoidal profiles allot equal time for each motion phase, while variable trapezoidal profiles use lengthened or shortened portions to complete strokes or movements more slowly or quickly. The 1⁄3-1⁄3-1⁄3 profile is the most power efficient for the linear applications we discuss here.

Linear motion



This Sanyo system puts shaft motors to use for precise force.

tal pause time. The first is the time for which load is stationary; the second is the amount of time it takes the load to come to a complete stop. Specifically, in servo-based systems, actual motor (or load) movement lags behind commanded controller operation. The distance of this lag is called thermal following error, or *TFE*. The amount of time that it takes the motor (with load) to move from TFE to target position (within a specified error band) is called settling time T_S. The amount of TFE largely depends upon deceleration rate and servo bandwidth and

f_0 = 50 Hz is a typical value.

ω_0 = Servo constant = $2\pi \cdot f_0$

Movement deceleration $A_d = V/T_d$

TFE = $4 \cdot A_d/\omega_0^2$

Next we must define the error band within which the system will settle. For our example here, let us call the point at which the proportional servoloop cannot move the motor (due to friction) the minimum following error, or *minFE*:

Servo stiffness $SS = [(M_c + M_L)\cdot\omega_0^2]/4$

$minFE = F_f / SS$

τ = Settling constant = $1/\omega_0$

Settling time $T_s = L_n(TFE/minFE)\ \tau$

If wait time is greater than 100 ms then the designer can get by without calculating settling time, as it will likely be less than waiting time.

Required force

The second step is to calculate required system force. Calculating this force for acceleration F_i (also known as force to overcome inertia) requires Newton's second law, $F = M \times A$. Because acceleration can also be expressed as velocity divided by time, we use:

$F = (M_c + M_L) \times V / T_a$

Next we calculate force required to overcome friction F_f — the force required to move the load. α is the angle of the move, vertical = 90° and horizontal = 0°, the friction coefficient of the sliding surfaces is μ, mass moved is $M_c + M_L$, F_r is run

Operation condition

These two forms can be helpful in collecting motion system information for applicaitons.

Item	Value	Notes
Load mass M_L in kg		Mass of moving system parts, less motor mass.
Load (thrust) force F_L in N		Thrust force is added to all segments of the motion profile. This is in addition to force needed to overcome mass, acceleration, and friction.
Run (preload) friction F_r in N		Preload force is considered in all moving segments of the motion profile. Consider all external forces that disturb the movement.
Moving motor mass M_c in kg		If unsure as to which motor is needed, start with a value of 1/10 of load mass.
Friction coefficient μ		
Incline angle α in degrees		0° is horizontal while 90° implies a vertical arrangement.
Available voltage V		Voltage is alternating current here.
Available current A in A_{rms}		
Maximum temperature in °C		Allowable temperature is for the overall design.

Some linear-shaft motor units include a slide guide and encoder for full actuator capabilities.

While this article illustrates sizing with only one segment, it is recommended that for the best linear shaft motor sizing, a complete cycle is used — including the stroke out and then back.

Item	Value
Stroke X in mm	
Velocity V in m/sec	
Acceleration time T_a in sec	
Acceleration time T_a in sec	
Deceleration time T_d in sec	
Settling time T_s in sec	
Waiting time T_w in sec	

friction, and g = 9.81 m/sec^2. For horizontal motion or arrangements in which an upward-sloping incline fights gravity:

$F_f = (M_c + M_L) \times g\,[\mu \times cos(\alpha) + sin(\alpha)] + F_r$

In contrast, for downward-incline moves that work with gravity:

$F_{fd} = (M_c + M_L) \times -g\,[\mu \times cos(\alpha) - sin(\alpha)] + F_r$

Example ❶
V = 760 mm/sec and T_a = 77 msec
M_L= 2 kg and M_c = 0.15 kg
Let us select an S160D forcer with F_i = (0.15 + 2) × (0.76 / 0.077) = 2.15 × 9.87 = 21.2 N
F_f = (0.15 + 2) × 9.81[sin(0) + 0.1 × cos(0)] + 0 Ff = 2.15 × 9.81× 0.1 + 0 = 2.1 N

Example ❷
V=625 mm/sec and T_a = 40 msec
M_L = 3 kg and M_c = 1.1 kg
Let us select an S250T shaft with a 100-mm stroke for F_i = (1.1 + 3) × (0.625 / 0.04) = 4.1 × 15.625 = 64.1 N
F_f = (1.1+ 3) × 9.81× [0.1 × cos(90) + sin(90)] + 0 = 4.1 × 9.81 × 1 + 0 = 40.2 N
F_{fd} = (1.1 + 3) × -9.81[0.1 × cos(90) - sin(90)] + 0 = 4.1 × -9.81 × -1 + 0 = 40.2 N

Force for each segment

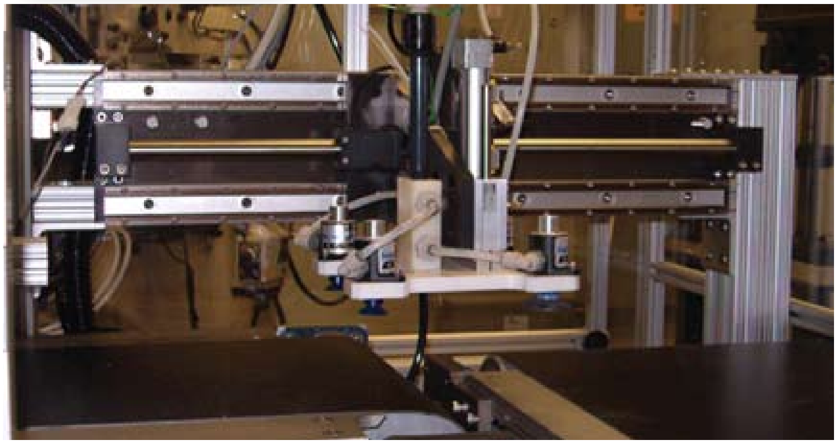
Force required by a system is different for each kind of motion that it executes; each consists of one or more of the following forces.

Load force F_L is a force that may stay constant. It is added when it works against motion and subtracted when it works with motion. In some cases (for example, against a spring) the force can increase. **Acceleration force** F_i only occurs during acceleration and deceleration. **Friction force** (F_f or F_{fd} above) occurs when load is in motion, and assists in deceleration. **Dwell force** F_d is made up of only gravitational forces, not seen in horizontal moves.

Acceleration: $F1 = F_i + F_f + F_L$
Continuous: $F2 = F_f + F_L$
Deceleration: $F3 = F_i - (F_f + F_L)$
Dwell: $F4 = (M_c + M_L)g \times [sin(\alpha)] + F_L$

Vertical or inclined moves use F_f for against-gravity moves and F_{fd} for with-gravity moves. Tip: When load force assists motion, change the sign of F_L from positive to negative, as it is impossible for a motor to produce a negative force. Force calculations that produce negative results indicate that the motor is producing that force opposite the direction of travel. For simplicity, any forces that are negative should be multiplied by -1 to make them positive.

Example ❶
F_i = 21.2 N ... F_f = 2.1 N ... F_L = 0 N
F1 = 21.2 + 2.1 + 0 = 23.3 N
F2 = 2.1 + 0 = 2.1 N
F3 = 21.2 – (2.1 + 0) = 19.1 N
F4 = (0.15 + 2) × 9.81 × [sin(0)] + 0



➲ **Motion in this Texwipe application utilizes linear shaft motors for their acceleration capabilities.**

F4 = 2.15 × 9.81× 0 + 0 = 0 N
Example ❷
F_i = 64.1 N ... F_f = 40.2 N ... F_{fd} = 40.2 N ... F_L = 0 N
Down: F1 = 64.1+40.2+0 = 104 N
F2 = 40.2 + 0 = 40.2 N
F3 = 64.1–(40.2+0) = 23.8 N
Up: F1 = 64.1–(40.2 + 0) = 23.8 N
F2 = 40.2 + 0 = 40.2 N
F3 = 64.1 + 40.2 + 0 = 104 N
F4 = (1.1 + 3)× *g* × [sin(90)] + 0
F4 = 4.1 × 9.81× 1 + 0 = 40.2 N

Effective (rms) and peak force

Peak force is the largest segment force. Effective (rms) force is the root mean square average of all the force segments — force required from the motor:

$$F_{eff} = \sqrt{\frac{\left(F1^2 \cdot T_a\right) + \left(F2^2 \cdot T_c\right) + \ldots \; \ldots \left(F3^2 \cdot T_d\right) + \left(F4^2\left[T_s + T_w\right]\right)}{T_a + T_c + T_d + T_s + T_w}}$$

Example ❶: F1 = 23.3 N ... F2 = 2.1 N ... F3 = 19.1 N ... F4 = 0 N ... T_a = T_b = 77 msec ... T_c = 487 msec ... T_s + T_w = 3,000 msec ... F_{eff} = 4.47 N ... F_{peak} = 23.3 N
Example ❷: **Down:** F1 = 104.3 N ... F2 = 40.2 N ... F3 = 23.9 N ... T_a =T_b = T_c = 40 msec ... T_s + T_w = 10 msec **Up:** F1 = 23.9 N ... F2 = 40.2 N ... F3 = 104.3 N ... F4 = 40.2 N ... T_s + T_w = 150 msec
F_{eff} = 57.01 N ... F_{peak} = 104.3 N

Motor selection

The third step is selecting a motor. Confirm that effective force F_{eff} is less than the motor's continuous rated force F_{rated} — and if the moving mass is larger than the value we first used for M_c then recalculate with the larger value.

Needed motor-specific information includes resistance *R*, inductance *L* in mH, force constant K_f in N/A_{rms}, back emf K_e in V/m/sec, magnetic pitch in N-N, and E_c in mm. In addition, two application-related variables are needed: peak velocity V_{peak} in m/sec, and peak force F_{peak} in N.

Example ❶: S160D Moving forcer F_{eff} = 4.44 N < F_{rated} - 10N ... Good ... F_{peak} = 23.3 N ... V_{peak}= 0.76 m/sec ... R = 21 _ L = 8.2 mH ... K_f = 16.1 N/Arms ... K_e = 5.37 V/m/sec E_c = 60 mm
Example ❷: S250T moving shaft with 100-mm stroke ... F_{eff} = 57.01 N < F_{rated} - 60 N ... Good ... F_{peak} = 104.3 N V_{peak}= 0.625 m/sec R = 12 _ L = 15 mH K_f = 46.88N/A_{rms} K_e = 15.63 V/m/sec E_c = 90 mm

Next, we calculate voltage and current required for the application, and confirm that the power source can handle it.

Resistance (hot) $R_{hot} = R \times 1.423$
Peak current $I_p = F_{peak} / K_f$
Voltage due to back emf $V_{bemf} = Ke \times V$
Voltage due to $I \times R$:
$V_{ir} = 1.225 \times R_{hot} \times I_p$
Voltage due to inductance:
$V_L = 7.695 \times V \times L \times I_p / E_c$
Required minimum bus voltage:
$V_{bus} = 1.15 \sqrt{(V_{bemf} + V_{ir})^2 + V_L{}^2}$

Example ❶: R_{hot} = 21×1.423 = 29.9 Ω ... I_p = 23.33/16.1 = 1.45 A ... V_{bemf} = 5.37× 0.76 = 4.08 V ... V_{ir} = 1.225 × 29.9 × 1.46 = 53.11 V ... V_L = 7.695 × 0.76 × 8.2 × 1.45/60 = 1.16 V ... V_{bus} = 1.15 √(4.08 + 53.11)^2 + 1.16^2 = 65.78 V
Example ❷: R_{hot} = 12×1.423 = 17.1 Ω ... Ip = 104.3 / 46.88 = 2.22 A ... V_{bemf} = 9.77 V ... V_{ir} = 1.225 × 17.1 × 2.22 = 46.5 V ... V_L = 7.695 × 0.625 × 15 × 2.22/90 = 1.78 V ... V_{bus} = 1.15 √(9.77 + 46.5)^2 + 1.782 = 64.74 V

Check the following five points: Is the initially used moving mass correct for this selected linear shaft motor? Is F_{rated} larger than F_{eff}? Is there a safety margin? (Certain manufacturers recommend that F_{eff} be no more then 80% of F_{rated} to allow for a 20% safety margin — 30% to 50% is optimal.) Is sufficient voltage supplied? Is current sufficient? If an engineer can answer yes to these questions, then this design portion is a success, and the selected linear shaft motor will work for the application at hand. The next step: Find a servo driver.

Driver selection

The fourth and final step in linear motor system design is to select a proper servo driver. At this point we have most of the information needed: application requirements (rms values), minimum ac bus voltage, and peak current. Now let us calculate continuous current and *peak-of-sine* or dc values. Continuous current is $I_{crms} = F_{eff} / K_f$ to peak (of sine) values, so Peak = rms × √2. For ac amplifiers we use rms values; for dc amplifiers we use peak-of-sines.

Example ❶: I_c = 4.47 / 16.1 = 0.278 A ... Vdc = 65.78 × √2 = 93.03 V ... I_{pp} = 1.45 × √2 = 2.05 A ... I_{cp} = 0.278 × _2 = 0.393 A
Example ❷: Ic = 57.01 / 46.88.1 = 1.22 A ... Vdc = 64.74 × √2 = 91.56 V ... I_{pp} = 2.22 × √2 = 3.14 A ... I_{cp} = 1.216 × √2 = 1.72 A

Sizing software in an Excel document from Nippon is available at motionsystemdesign.com. For more information, visit nipponpulse.com or call (540) 633-1677.